

Unique Paper Code : 22411402\_OC  
 Name of the Paper : Business Mathematics  
 Name of the Course : B.Com (H) CBCS  
 Semester : IV  
 Duration : 3 hours  
 Maximum Marks : 75 Marks

**Instructions:**

- 1) This paper contains six questions in total, attempt any four questions.
- 2) All questions carry equal marks.

Q1 a) Mr. Ram has divided ₹35,000 into three investments. Part of the money is invested in fixed deposit with an annual rate of interest of 6%, part in 7% annual yield bonds, and the remainder in a business. In 2020, when he lost 6% of the money that he invested in that business, his net income from all the three investments was ₹660. If he invested ₹3,000 more in the business than in the fixed deposit, how much was invested in each? Use matrix method.

b) Suppose the inter-industry flow of the products of two industries is given as under:

| Production Sector | Consumption Sector |    | Domestic Demand | Total Output |
|-------------------|--------------------|----|-----------------|--------------|
|                   | X                  | Y  |                 |              |
| X                 | 20                 | 90 | 90              | 200          |
| Y                 | 30                 | 15 | 105             | 150          |
| Labour (hours)    | 50                 | 30 |                 |              |

- i) Determine the technology matrix and test Simon-Hawkins conditions for the viability of the system.
- ii) If the domestic demand changes to 144 and 72 units respectively, what should be the gross output of each sector in order to meet the demands?
- iii) If the total labour available is 100 hours, is the solution feasible?
- iv) Find the total value added if the wage rate is ₹150.

Q2 A firm uses three machines in the manufacture of three products. Each unit of product A requires 3 hours on machine I, 2 hours on machine II and one hour on machine III. Each unit of product B requires 4 hours on machine I, one hour on machine II and 2 hours on machine III, while each unit of product C requires 2 hours on machine I, 3 hours on machine II and 3

hours on machine III. The contribution margin of the three products is ₹15, ₹30 and ₹20 per unit respectively. The machine hours available on three machines are 120, 80 and 150 respectively.

- i) Formulate the above problem as a linear programming problem.
- ii) Obtain optimal solution to the problem by using the simplex method. Which of the three products shall not be produced by the firm? Why?
- iii) Write the dual of the above problem.
- iv) What are the shadow prices of the machine hours?
- v) Is the optimal solution degenerate?

Q3 a) Suppose that  $x$  years from now, one investment plan will be generating profit at the rate of  $R'_1(x) = 100 + x^2$  rupees per year, while a second plan will be generation profit at the rate  $R'_2(x) = 220 + 2x$  rupees per year.

- i) For how many years will the second plan be the more profitable?
- ii) How much excess profit will you earn if you invest in the second plan instead of the first plan for the period of time in part (i).

b) A production function is given by  $Q = 24L^{3/8}K^{3/8}$ , where  $L$  are units of labour and  $K$  units of capital.

- i) Find the behaviour of the marginal product of each factor.
- ii) What is the nature of returns to scale.
- iii) Is the total output exhausted if each factor is paid a price equal to its marginal product?

c) A bicycle manufacturer buys 2500 tyres a year from a distributor. The ordering fee is ₹20 per shipment plus a ₹9 insurance charge for each tyre ordered. The storage cost is ₹10 per tyre per year. Suppose that the tyres are used at a constant rate throughout the year and that each shipment arrives just as the preceding shipment is being used up. Find the number of tyres that the manufacturer should order each time to minimize the total inventory cost.

Q4 a) Suppose that the demand equation for a certain commodity is  $x = a/p^m$  where  $a$  and  $m$  are positive constants. Show that the elasticity of demand is equal to  $m$  for all values of  $p$

b) The demand functions for two products are

$$p_1 = 12 - 2x_1 \qquad p_2 = 20 - x_2$$

Where  $P_1$  and  $P_2$  are the respective prices for each product and  $x_1$  and  $x_2$  are the respective amounts of each sold. Suppose the joint cost function is:

$$C = x_1^2 + 2x_1x_2 + 2x_2^2$$

Find the revenue function and the profit function. Determine the number of units and price

of the two products that will maximise profit. What is the maximum profit?

- c) ABC company wants to buy an energy-saving machine which will reduce its consumption of fuel. The machine will cost ₹32,000. It is estimated that saving from using the machine will be at a rate of  $S(t)$  rupees per year, where  $S(t) = 20,000 e^{-0.5t}$  and  $t$  equals time measured in years. Determine how long it will take for the firm to recover the cost of the machine.

Q5 a) Mr. X borrowed ₹15,00,000 from bank and decided to repay it by equal quarterly instalments over a period of 5 years. The bank charges interest at 6% p.a. compounded quarterly. Calculate the amount of quarterly instalment. Also find the principal contained in 5<sup>th</sup> instalment.

- b) Find the amount that an investor will receive after investing ₹15,000 for 7 years when the rate of interest is 6% p.a. compounded quarterly for first 3 years, 7% effective for next 2 years and 7.5% p.a. compounded continuously thereafter. Find the excess or shortage of fund/amount if he requires ₹23,000 for purchasing a machine after 7 years.
- c) A sinking fund is created for redemption of debentures of ₹1,50,000 at the end of 12 years. How much money should be provided out of profits at the end of each quarter for the sinking fund, if the investment can earn interest at 4.5% p.a. compounded quarterly?

Q6 a) A firm has available two kinds of fruit juices (pineapple and orange juice). These are mixed and the two types of mixtures are obtained which are sold as soft drinks  $A$  and  $B$ . 1 tin of  $A$  needs 4 litres of pineapple juice and 1 litre of orange juice. 1 tin of  $B$  needs 2 litres of pineapple and 3 litres of orange juice. The firm has available only 46 litres of pineapple juice and 24 litres of orange juice. Each tin of  $A$  and  $B$  is sold at a profit of ₹4 and ₹3 respectively. How many tins of  $A$  and  $B$  should the firm produce to maximize profit? Formulate the problem as a Linear Programming Problem and solve it by graphic method.

- b) The demand equation for a monopolist is  $p = 200 - 3x$ , and the cost function is  $C(x) = 75 + 80x - x^2$ .
- Determine the level of output and the corresponding price that maximize the profit.
  - If the government imposes a tax of ₹  $t$  per unit. Determine the new output and the price that maximizes the monopolist's profit.
  - Determine the value of  $t$  that will maximize the tax revenue and the total tax revenue received by the government.